Diffusion rendering of black ink paintings using new paper and ink models
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Abstract
A practical technique is presented for efficiently rendering oriental black ink paintings with realistic diffusion effects, being based on models which simulate a variety of paper types and black ink properties. Strokes can be generated by bristles or elastic brush models simulating those actually used in oriental black ink painting. Incorporation of paper modeling allows users to draw on a wide variety of papers having either randomly generated "fiber meshes or uniform meshes commonly found in fabrics which are also frequently used. This accordingly provides a unique platform for real-time dynamic simulation of realistic ink diffusion in that the resultant diffusion images reflect the texture and the global change of gray tones as well as local temporal variations specific to a particular paper. Examples are presented demonstrating the capability of the proposed technique for handling different ink and paper properties.

Keywords: Diffusion rendering; Soft brush; Paper modeling; Fiber mesh; Oriental black ink painting

1. Introduction
The traditional Asian art form of black ink painting took root more than 3000 years ago [1,2]. Displaying dynamic formations, delicate gray shading tones, and impressive textures with only a few strokes have attracted numerous researchers in computer-simulation attempts. While the technique may initially appear to be simple, the abundant number of expressive rendering effects, and their subsequent simulation, serve to arouse increasing research interest among computer scientists.

Black ink rendering is non-photo realistic rendering [3–7] which stands in contrast to conventional computer graphics rendering methods that strive for photo realism, e.g., ray tracing and radiosity. In fact, it is even distinct compared to general non-photo realistic rendering such as digital painting, picture retouching, or illustration generation; methods which involve a wide range of colors, rich editing functions, and various brush patterns, but which do not involve the brush models, special paper, or ink used in black ink painting.

The brush used in black ink painting has long, star-ched natural bristles which narrow to a pointed tip. The bristles, typically made from the hair of rabbits, martins, badgers, horses, or deer, move with elastic deformation on the paper such that 3D visual information is provided to the artist; a crucial feature to timely guide the brush direction. The paper used in black ink painting differs from that typically used in watercolor painting, being much thinner and more textured. Moreover, it has little sheen and is quite absorbent; a feature which allows the painted fluid to more easily flow along its fibers. The employed black ink is a dilute mixture of water and colloidal black carbon particles in which the carbon particles are much smaller than those of watercolor paints such that they can diffuse into paper along with the absorbed water. One early attempt to simulate black ink painting applied conventional 2D rendering techniques, i.e., in order to synthesize the stroke produced using hair bristle brushes, the outline of brush strokes was described as a sequence of connected Bézier or B-spline curves [8,9]. The curves are developed using 2D drafting software in conjunction with mouse or pen input.
and scaling/editing control points. Produced strokes are filled with image patterns incorporating a set of shade variation effects, after which they can be modified by reapplying the same 2D drafting and filling functions. While this approach gives precise, smooth outlines with variations in shading, the mode of access is limited by the employed drawing process due to the fact that stroke editing is restricted to altering control points that determine stroke boundaries.

Other attempts were also made to apply photo retouching techniques or paint tools developed for watercolor painting to black ink painting. Haebleris used simple primitives such as points, lines, triangles, and 2D patterns on a photo retouching system to generate or transform strokes [10], while Curtis introduced a model which simulated watercolor painting and black ink rendering effects [11].

Many commercial photo retouching softwares and digital paint tools are currently available. Adobe Photoshop, a well-known commercial product, incorporates a Gaussian blurring filter that allows users to paint soft-edged strokes or blur a stroke. Fractal Design Painter is another popular commercial system that has many effective paint tools for simulating various painting materials and touch effects [12]. Its brush pallet has several types of pixel brushes that simulate the art effects produced by crayons, chalk, an airbrush, watercolors, and oil paints; while its art material pallet provides many types of paper texture patterns for representing the roughness of a paper’s surface. The software, however, is limited in its ability to obtain realistic black ink painting effects in that the resulting strokes are blurred globally and the color around the edges of the strokes is lighter than the inside part, simulating feathering effects of watercolor rather than of black ink. In black ink rendering, the rendering of individual strokes changes delicately according to the ink status, speed, and movement of the brush.

Taken together, it becomes clear that the inherent restrictions in commercial software for generating black ink painting are mainly due to the special characteristics of the particular materials used. Researchers attempting to simulate oriental black ink painting have realized such requirements and accordingly considered the possibility of modeling brushes and papers [13,14] or to instead use real tools [15,16]. Strassman proposed a physical model for representing brush strokes and rendering, i.e., the brush is considered as a 1D array of bristles and moved such that it is always perpendicular to the path of the stroke defined by a set of position and pressure parameters in which pressure determines stroke width [13]. While this approach does indeed enable users to produce brush strokes by moving a brush model, the brush model itself could still be improved, especially with regard to black ink painting. That is, as the brushes are simply 2D patterns like rubber stamps, users cannot see how the shape of the bristles is transformed as bristles are pulled, turned, pressed down, or lifted up; visual information which is essential for users to have so as to better control brush movement. Moreover, because such imaginary bristles do not bend, the timing in which pressure is exerted on the brush is quite different compared to that of a real brush; hence users cannot utilize acquired painting skills developed over a lifetime. Most recently, we proposed a new brush model that overcomes the inherent drawbacks of Strassman’s brush model, i.e., we developed 3D “soft” brushes in which the shape of the bristles varies dynamically in response to the forces imposed on it by the paper [17].

Although orientation and density of fibers, as well as brush shape, are required to perform black ink rendering, only the local ground roughness of the paper is required for modeling the spreading of watercolor. In 1991, Guo and Kunii proposed a basic fiber mesh paper model for synthesizing black ink diffusion [18]. This approach produced diffusion images based on a paper model, yet problems arise in that their model considered randomly distributed fibers, but did not take into account the material and texture of the paper, nor was the dynamic process of ink and image density variation included.

Here, we present a complete black ink diffusion rendering method which considers the characteristics of black ink and also incorporates a random or regular fiber mesh structured paper model. Using this new method in conjunction with our previously proposed brush model [17] improves black ink diffusion rendering approaches in several aspects: (1) the user can select a wide variety of paper textures, (2) realistic diffusion images can be drawn reflecting the paper material and ink characteristics, (3) the efficient and novel rendering algorithm is applicable to synthesize diffusion effects for other art styles as well.

Section 2 provides a brief explanation of ink diffusion phenomena, while Section 3 gives an overview of the soft brush used for creating brush strokes. Section 4 then discusses the modeling of paper and parameters that define the geometry of the fiber mesh, after which Section 5 describes the diffusion rendering algorithm based on the paper model. Typical diffusion images created with these techniques are presented in Section 6, being followed by Section 7 in which diffusion rendering is applied to a variety of paper types.

2. Ink diffusion phenomena

Diffusion of the painted fluid is perhaps the most admired feature of this unique art form. A type of halo appearing around the original stroke adds a mysterious touch, being caused by letting ink spread beyond the stroke’s original border, while ink seeping into special paper with high absorbency creates a feathery, blurred
edge. These diffusion features represent complex physical phenomena which cannot be completely simulated by simple degradation functions, fractals, or texture mapping techniques, since purely mathematical methods generally result in flatly blurred images which are different compared to realistic diffusion images. Moreover, the computational burden becomes excessive when they are applied to strokes having complex shapes.

Development of an appropriate model for simulating ink diffusion requires attention to be focused on the occurring physical mechanism. The typical paper is a mesh of fibers in which small holes or spaces between the fibers act as thin capillary tubes for carrying water away from the initial area. The carbon particles float and move in this liquid due to collisions with molecules of water and carbon particles. Accordingly, an appropriate simulation of ink diffusion must include careful modeling of the following aspects:

1. fiber material and mesh structure of the paper,
2. ink granules,
3. quantity of water,
4. ink density at a point,
5. states of the surrounding points.

Depending on which type of paper is used, the ink absorbency, diffusion directions, and diffusion patterns will vary. The type of fiber material and fabrication of fibers are responsible for this. Three paper types are commonly used in black ink painting, i.e., rice paper, egg paper, silk cloth. As shown in Fig. 1, diffusion is strong in rice paper but not in egg paper. Also, rice paper produces diffusion scattering outwards in all directions, whereas that in silk is oriented in two directions perpendicular to each other.

The granule size of black ink particles affects the color intensity of the boundary between the original stroke and its diffusion image (Fig. 2). Black ink is acquired by rubbing an inkstick over an inkstone which is a shallow slate dish with a reservoir at one end. When the acquired ink is coarse, i.e., consisting of small and large particles, the boundary between the original stroke and diffusion area looks clear; a phenomenon that occurs because only those particles whose granule size is smaller than the space between fibers can seep into the mesh along with water. Particles whose granule size is bigger than the space remain at the initial position. This phenomenon is referred as a “filtering effect” of the fiber mesh. On the other hand, if the ink is homogeneous and consists of small, uniform particles, most ink particles move with water unhindered by the fibers, such that a continuous and smooth intensity change appears across the diffusion area.

Ink particles are carried away from the initial area by water, which as it passes along the fibers is absorbed by the fibers while any remaining water continuously flows along the fibers until completely absorbed. The quantity of water, and not the density of ink, accordingly determines the span of the diffusion image or the number of diffusion steps.

Under the influence of the motion of water molecules, suspended ink particles move in a manner called Brownian motion [19,20]. A mixture of two inks having different densities will produce an irreversible diffusion process in which ink particles transfer from the ink with higher density to that with lower density. The density of liquid ink surrounding a particular point will therefore determine the direction in which diffusion takes place.

3. Producing brush strokes

Brush strokes are produced by directly moving our soft-brush model over the new paper model. As with a real brush, the model’s bristles possess elastic properties such that application of an external force will deform them, though they nearly resume their initial form after the removal of the force; a technique eliminating unnatural spline curve editing required by the conventional 2D drafting approach.

In response to brush movement along a path, boundary lines of the stroke, being smoothed with spline curves, are automatically generated. For rendering inside the

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Fig. 1. Diffusion images generated by ink dropped on (a) rice Paper (Washi in Japanese), (b) egg Paper (Torinoko), and (c) silk cloth (Eginu).
stroke, two methods are available (Fig. 3). The first is “simple-trajectory rendering” in which a “footprint” of all the bristles is drawn on the paper, while the second method is “boundary-shading rendering” in which the boundary line for each stroke segment is calculated and the area within the boundary is subsequently smoothed. As each method has its own merits, either can be selected by the user as desired. When the former is used, rendering of strokes is straightforward in that the paper pixels in contact with each bristle are simply painted according to the ink quantity remaining in the bristle; while in the latter method, the boundary of the stroke segment and the amount of ink within are both iteratively calculated.

As shown in Fig. 4, a stroke consists of a sequence of polygons or “sections” surrounded by two boundary lines. The two boundary lines are represented as a list of edge-nodes, where for each node, the position, color density, and liquid ink property of the left and right edge of the stroke are computed using the bristles that swept over that particular point, i.e.

\[
P(t) = \{ (\text{position}(t), \text{pressure}(t)) | t = 0, \ldots, n \} ,
\]

\[
L(t), R(t) = \{ (\text{position}(t), \text{density}(t), \text{ink}(t)) | t = 0, \ldots, n \} ,
\]

\[
\text{NODE}(t) = \{ (L(t), R(t)) | t = 0, \ldots, n \} ,
\]

\[
\text{SECTION}(t) = \{ \text{NODE}(t), \text{NODE}(t + 1) | t = 0, \ldots, n - 1 \} .
\]

The information saved in the nodes is transferred to the “initial diffusion front” as discussed in Section 5.1.

4. Modeling the paper

While handmade paper is most commonly used in oriental black ink painting, fabric is another material frequently selected because paper is not as durable as cloth, nor is it commonly available in large sizes. Moreover, painting on cloth allows the final artwork to be treasured for many years, or in other cases, used as a flag or sign. Because paper was a luxury in ancient days, many people practiced painting on fabric to allow washing out and starting over.

Materials to be drawn on typically consist of a regularly or irregularly distributed fiber mesh. Rice paper consists of a mesh of randomly positioned fibers, while silk
fibers are uniformly aligned due to being made by weaving. Here, a randomly distributed fibrous mesh and uniform fabric structure are both modeled, with each type of material being referred to as “paper”.

4.1. Paper with randomly distributed fibers

Modeling paper comprised of a random fiber network was proposed by Kallmes and Corte as early as in 1960 [21,22]. Computer simulations have often been used in paper manufacturing research, and later applied to computer-based painting systems as well. Regarding oriental black ink painting, Guo and Kunii simulated a paper consisting of two-dimensional fiber networks, i.e., they divided the entire field into regions in which each contained curved lines drawn with random orientation [18]. Details of the division method and fiber shape were not reported, however.

We represent each single fiber as a curved line segment using a sine function which is smooth, computationally simple, and symmetric about the origin (Fig. 5). The properties of fiber distribution are such that the local position and orientation of fibers vary randomly whereas the global property of fiber distribution is uniform. A fiber mesh is accordingly constructed with these properties by dividing the paper plane into square regions and distributing the fibers according to a rule; i.e., for each region the average fiber density is the same, although within each region, fiber centers and orientations vary randomly.

We selected a square region because it is the simplest geometric shape permitting complete covering of a rectangular sheet of paper; unlike circles which would be simpler but do not cover the whole area [23]. Regarding the optimal size of the region, which is an important parameter to obtain a balance of the “global homogeneous and local random” property of the paper, we selected double the length of a fiber as the length of a side; a size in which each region can contain segments of fibers that pass through, terminate in, or lie entirely within.

A peculiar characteristic of paper with randomly distributed fibers is that the regions at the border of the paper will tend to have a low fiber density if the paper field is blindly subdivided into regions in which the centers of fibers can randomly move within each region. As this problem has never previously been dealt with, we mitigate it by assuming that on the surface of the paper where fibers are to be generated there exists a “fiber generation plane” that is sufficiently wider than the paper (Fig. 5). Segments of fibers contained in the overlapped area of the paper plane and the fiber generation plane are therefore considered to be part of the paper.

A fiber is defined as

\[ y = c \sin \frac{2\pi x}{l}, \quad -\frac{l}{2} \leq x \leq \frac{l}{2}, \]

where \( c \) is the curvature of a fiber and \( l \) is its length. Fibers with various curvatures and lengths can be obtained by changing these values. A fiber can be oriented and located at a new position by applying 2D transformations to the coordinate vectors.

To ensure the local fiber density and orientation are random while the global density and orientation are uniform, the large paper field is evenly subdivided into small rectangular subareas having the same average fiber density, although the fiber distribution varies randomly from one subarea to another.

Once fibers are distributed over the paper field, a fiber mesh structure is determined at every pixel of the paper such that each one can be represented as a data parameter.

![Fig. 5. A fiber generation plane covers the paper (paper plane) and is subdivided into square regions in which fibers are generated in consideration of global uniformity and local randomness.](image-url)
structure, i.e.

def class Papel {
    int x, y;  // coordinate
    short captub[8];  // capillary tubes
    captub_sum;
    short cross_pts;  // number of cross points
    short water, granule, diff_type;  // liquid ink characteristics
}

where the papel $P$ has eight neighbor papels $P_0, P_1, \ldots, P_7$, $captub[i]$ denotes the number of fibers passing $P$ and $P_i$, and $cross$ _pts_ denotes the number of cross points where two or more fibers cross each other (Fig. 6). It is assumed that the number of capillary tubes connecting $P$ and $P_i$ is proportional to the number of fibers passing $P$ and $P_i$. The relation between cross points and the papel’s absorbency will be explained in Section 5.2.2.

4.2. Paper with regular fiber mesh pattern

In the employed system, a fiber mesh with regular patterns can be designed using two methods. The first involves determining the shape and distribution of fibers by manipulating various parameters, i.e., the curvature of a single fiber $c$, length of a fiber $l$, fiber density or the number of fibers $n_i$ fiber location $(x_1, y_1)$, or fiber orientation $\theta$. The second method involves determining fiber mesh information from a particular texture. That is, a rectangular array of the texture is mapped onto the paper by flat tiling, after which fiber mesh information at each papel is calculated based on the shading of its neighboring papels.

5. Ink diffusion rendering

The above-described fiber mesh structure of the paper provides information about how each point on the paper is connected to its neighboring points. The next step in simulating diffusion rendering is to determine the schema simulating the point-to-point flow of ink through the fiber mesh.

5.1. Ink flow schema

We developed a “wave” schema for representing how ink flows through a fiber mesh. Diffusion is considered to originate from the “boundary points” of strokes, being analogous to the outward-moving circular waves produced when an object is thrown into a lake. In other words, water oscillates up and down during wave movement and the papels of paper diffuse color when the “diffusion wave” arrives.

At any particular moment, the points at the edge of profile of the diffusion area are collectively termed as the “diffusion front” such that the “boundary points” are the “initial diffusion front”. The diffusion process accordingly involves using the current diffusion front to successively determine the next diffusion front as a time sequence represented by step counter changes from zero to $n$. Fig. 7 depicts the main principles of ink flow under the proposed wave schema. Important aspects are as follows:

1. From a point $P$ at the current diffusion front, ink can only flow to point $P'$ if it is connected to $P$ and is dried.
2. Point $P'$ absorbs some amount of liquid ink before it transports ink to other points.
3. The ink absorbed at $P'$ evaporates after unit time $\Delta t$, where it is assumed that $\Delta t = 2$.

In our wave schema, (i) the ink density at the diffusion front is determined before the diffusion wave continues on, (ii) the points covered by the diffusion front will not
be included in the next diffusion front for a short period of time; hence the number of points involved in the diffusion process can only linearly increase over time, and (iii) the ink cannot flow backward because the diffusion wave travels only outward. Moreover, the wave schema allows diffusion by overlapped strokes because ink on prior strokes is considered to evaporate after a set time elapses.

5.2. Papel intensity decision schema

Regarding the diffusion rendering algorithm, the above procedure was implemented such that the next diffusion front is determined from the current diffusion front, while the color intensity of papels in the next diffusion front is determined from the color intensity of papels in the current diffusion front. In calculating the image intensity of the diffusion image, two factors must be considered: (1) the global and continuous change of intensity moving from the boundary of stroke toward the edge of diffusion image and (2) the local variance of intensity. The first factor depends on a fiber’s water-adhesive property, geometry of the fibers, mesh structure, density of liquid ink at the source points, and the value of step counter, whereas the second one depends on the local fiber density and the configuration in which the fiber mesh is interlaced.

5.2.1. Global change of color intensity

Direct observation of any diffusion image reveals that the density of ink decreases during the diffusion process due to the deposition of colloidal ink particles onto the fibers. However, since ink diffusion is a complex hydrodynamic percolation phenomenon caused by absorbent fibers, mesh structure, and special characteristics of the ink, it is difficult to formulate a mathematical equation describing ink dynamics for a specific type of paper.

To actually observe changes in ink density, we performed an experiment on real circular diffusion images generated on two different types of paper. Briefly, after an image was divided like a pie into four equal parts, all the papels of one part were plotted giving the color density as a function of radial distance from the center of the image as shown in Fig. 8. The indicated green skeleton lines were obtained by averaging the color density of the papels having the same distance value. Then, based on the geometric feature of the skeleton line, the diffusion area is divided into two zones: (1) the “diminish zone” where the ink density gradually decreases, and (2) the ensuing “vanish zone” where the ink density is weak and fluctuates. The whole rendered area can now be divided into the “stroke area”, where ink was directly applied on the paper, and the “diffusion area” that appears as the result of diffusion. The diffusion area is divided again into the “diminish zone” and “vanish zone” (Fig. 9).

The shape of the skeleton line and the deviation of papels around it will markedly vary from paper to paper. Using mulberry rice paper as an example, as shown in Fig. 8, although the mulberry paper shows a lower density of ink between the stroke area and diffusion area, the scale of fluctuations in the vanish zone is relatively higher. Such differences usually produce a diffusion image with strong impression as on mulberry paper, whereas that on rice paper is soft and light.

These images reveal that it is inappropriate to predict the global change in the intensity of the diffusion image based solely on the fiber mesh structure, or even by applying in general the same mathematical equations to describe color intensity of a point. This is true because two fiber meshes having the same local fiber density and interlacing structure can produce different diffusion patterns according to the fiber material used. The local diffusion direction is typically determined by the local geometric structure of the fiber mesh, while the global change of image intensity is determined by numerous interacting parameters including fiber density, evenness, orientation, type of fiber and interlacing, length, thickness, adhesive force, and flow dynamics of ink.

In order to synthesize a realistic diffusion image reflecting a paper’s distinct properties as closely as possible without employing complex equations, data on the global change of image density is obtained from an actual diffusion image. The skeleton line in Fig. 8 is smoothed using a spline curve to obtain the differentiable curve shown in Fig. 10(a), which is termed the “global diffusion” (GD) function. A differential curve of the GD function is termed the “global diffusion differential” (GDD) function, being that from which the diffusion rendering algorithm derives the papel intensity value (Fig. 10(b)). The relationship between a GD function G and its GDD-function G' is expressed as

\[ G(s) = \int_0^s G'(t) \, ds \]  

\[ = G(s - 1) + G'(t) \text{ for some } t \in [s - 1, s] \]  

\[ \approx G(s - 1) + G'(s - 1), \text{ if } |G(s) - G(s - 1)| \text{ is sufficiently small.} \]

5.2.2. Local variation in color intensity

A diffusion image shows different local intensity according to the local fiber density and structure of interlacing fibers. The papels scattered around the skeleton line in Fig. 8 can be explained on the basis of local variance in color intensity. At any point, the higher the fiber density the more liquid ink is absorbed such that a higher intensity is achieved. The orientation and interlacing pattern of fibers are also reflected in the diffusion image because ink flows along the fibers.

Ink absorbency is described using the following proposition.
Proposition 1 (Ink-absorbency). The ink absorption at paper $P$ is approximately proportional to the number of cross points in the paper.

Proof. For paper $P$, ink absorbency is evaluated as the ratio of the wet area covered by the ink to the area of the paper. Fig. 11(b) shows a case in which $n$ fibers cross at a point with orientation intervals that are ideally even. The area covered by ink, i.e., that indicated as a shadow region, is

$$A_n(P) = \sum_{i=1}^{n} S_i$$

$$= 2n^2 \sin \frac{\theta}{2}$$

$$= n^2 \sin \frac{\pi}{n} \quad n \geq 2$$
Using Taylor’s formula, Eq. (6) can be expressed as

\[
A_{\alpha}(P) = nr^2 \left( \frac{\pi}{n} + \sum_{i=0}^{n} \left( \frac{-1}{2n+1} \left( \frac{\pi}{n^2} \right)^{2n+1} \right) \right)
\]

(7)

\[
= r^2 \pi + r^2 \left( \sum_{i=0}^{n} \left( \frac{-1}{2n+1} \left( \frac{\pi}{n^2} \right)^{2n+1} \right) \right), \quad n \geq 2, \quad (8)
\]

where the remainder term can be ignored for the approximation of \( A_{\alpha}(P) \). It is therefore clear that \( A_{\alpha}(P) \) can be assumed to be constant regardless of the value of \( n \).

The above approximation is reasonable because fibers are very thin and predominately lie on the same 2D surface. The width of a fiber is in general about 1/100 of its length; hence the area of ink in a papel having a single fiber passing through it (Fig. 11(a)) is small \( (A_1(P) = \sqrt{0.0125 l}) \) such that it can be considered negligible.

Fig. 9. The rendered area can be divided into stroke and diffusion areas in which the latter is again divided into diminish and vanish zones.

Fig. 10. The (a) global diffusion function and (b) global diffusion differential function employed for rice paper.

Fig. 11. Papels in which (a) one fiber is present, (b) \( n \) fibers cross at a point, and (c) three cross points are present.
Proposition 2 (Papel intensity). The intensity value at papel P is determined by $D(P)$, $A(P)$, and $W(P)$, where $D(P)$, $A(P)$, and $W(P)$ are, respectively, the value of ink density, ink absorbency, and amount of liquid ink at the papel.

The mass of ink particles flowing into $P$ is $D(P) W(P)$, while the mass of absorbed ink particles that remain in the papel after water evaporates is $D(P) A(P) W(P)$.

5.3. Diffusion rendering algorithm

The following ink diffusion algorithms were constructed based on the principles of ink flow and papel intensity discussed thus far. When a short stroke is produced quickly, the diffusion process starts at the points along the boundary of the brush stroke almost at the same moment. However, for a long and slowly drawn brush stroke, diffusion in old sections may start earlier than in new sections, and the ink within old sections may even dry up. Our model allows users to choose either “stroke-unit” or “section-unit” rendering. With the former, diffusion rendering starts at all papels of the boundary line at the same moment. With the latter, however, the diffusion process starts in sequence among sections of a stroke from old to new.

Algorithm 1 (Diffusion front). To determine the sequence of diffusion fronts:

1. In stroke-unit rendering, find the points on the original stroke boundary, and save them in a queue termed the “diffusion front queue”. In section-unit rendering, choose the oldest section and save the boundary points in the diffusion front queue. The step counter is set to 0 because the source points form the initial diffusion front.
2. For each point $P$ of the diffusion front queue, determine its “next” points such that the next points of a source point are neighboring points connected by one or more capillary tubes that contain no water.
3. Insert all the next points into the next diffusion front queue. In section-unit rendering, choose the next oldest section if it exists and add the boundary points into the next diffusion front queue.
4. Using the below papel intensity algorithm, determine the ink density at each point of the next diffusion front queue according to the original intensity value of the point, the value of its source point, and the step counter.
5. If the next queue is empty or all the points in it have no remaining ink, terminate the process; otherwise, let the next diffusion front queue be the diffusion front queue, increase the step counter by one, and repeat the procedure from Step 2.

Algorithm 2 (Papel intensity). To calculate intensity value for each next point $P'$ in the next diffusion front:

1. The color intensity at $P'$ is evaluated as $D(P')A(P')W(P')$ using

$$D(P') = D(P) + G'(P)$$ using Eq. (3)

$$A(P') = C(P')/C_{average}$$, where $C(P')$ indicates the value of cross_pts at $P'$

$$W(P') = W(P) - 1$$.

2. If the step counter = 0 and the diffusion front is about to move forward, then the filtering effect explained in Section 2 is applied. According to the ink granule size coefficient $g$ normalized into $[0.0, 1.0]$, the color intensity calculated at Step 1 can be lowered, i.e.,

$$\rho(P') = \min(1 - g) \rho(P), D(P') A(P') W(P')$$.

3. If the diffusion front reaches the vanish zone, since the amount of remaining water is small, the diffusion process stops or continues irregularly from point to point according to the local capillary structure of fiber mesh. It is assumed that when $M_i \geq 2$, ink flows from $P$ to $P_i$; otherwise not. Note that in the diminish zone even a single fiber connection ($M_i = 1$) allows ink to flow from $P$ to $P_i$.

6. Variety of ink diffusions

Figs. 12–15 show the results of diffusion rendering carried out on a simple leaf-shaped stroke in which the amount of water, ink parameters, or type of paper are, respectively, varied.

Fig. 12(a) shows a ink painting drawn with a fairly dry brush ($W = 2$), while Fig. 12(b) is one drawn with a sufficiently wet one ($W = 10$). Note that the former case leads to diffusion over a wide area in contrast to the latter one in which all the water is absorbed by fibers before the diffusion wave can spread beyond the boundary of stroke.

![Fig. 12. Black ink diffusion renderings using ink containing (a) only a limited amount of water ($W = 2$) producing weak diffusion and (b) an abundant amount of water ($W = 10$) producing strong diffusion.](image)
Fig. 13. Black ink diffusion rendering using ink consisting of (a) coarse particles \((q = 0.7)\) which produce a distinct stroke border line and (b) small, homogeneous particles \((q = 0.0)\) which produce smooth color intensity changes across the stroke border line.

The granule size of ink particles affects the color intensity of the border line which appears along the edge of the initial diffusion front. The coefficient of ink particle coarseness is represented as a real number between zero and one. In Fig. 13(a), ink particles are coarse \((q = 0.7)\) such that the ink contains small and large particles which produce an observable change of color intensity along the border line. This occurs because only sufficiently small ink particles can flow (seep) into the fiber mesh along with water; a phenomenon referred to as the “filtering effect” of the fiber mesh. As shown in Fig. 13(b), decreasing the coefficient of ink particle coarseness to 0.0, which represents homogeneous ink consisting of small, uniform ink particles, produces smooth color intensity variations across the border line. Varying the fiber density and type of material also produces delicate variations as shown in Fig. 14(a), where the number of average fibers per region is doubled to obtain a denser paper. Note that the image has markedly darker rendering, especially in comparison with Fig. 14(b) where reducing the number of fibers results in the presence of small white spots due to a looser fabric.

More interesting and dynamic diffusion images can be created using papers with special textile patterns, or by exerting external forces. When painting on a cloth woven in a specific pattern, both inside the stroke and the ink diffusion image reflect the pattern, although the points where no fibers exist will not be rendered. Fig. 15(a) shows a simulated black ink painting of a leaf drawn on a cloth woven in a “mandala” pattern. If a force which is not normal to the surface of the paper is exerted on the ink, the adhesive force of water causing capillarity will change. The net force or vector sum of the external force and adhesion force of water molecules tangent to the capillary tube will act on the liquid. In Fig. 15(b), the paper was supposed to be situated vertically during painting, with the resultant diffusion area appearing to have slid downward due to gravity.

Fig. 14. Black ink diffusion rendering on (a) dense paper which produces a dark image and (b) loosely fabricated cloth which produces small white spots due to spatial separation between fibers.

Fig. 15. Black ink diffusion rendering on (a) cloth woven in a Mandala pattern and (b) paper situated vertically during painting producing the effect of sagging due to gravity.

Fig. 16. Black ink image of a dragon fly being painted with a soft brush.
7. Discussion

The described diffusion rendering technique generates realistic diffusion images using models that account for fiber mesh structured paper and ink flow. These techniques allow computer simulating black ink painting and calligraphy, as well as rendering general graphic images when combined with an edge detection algorithm. Although the proposed method can be applied to any existing painting system, it is considered most effective when brush strokes are generated using a soft brush.

Fig. 16 shows a simulated black ink painting of a dragon fly generated using a soft brush, while Fig. 17 shows corresponding simulations generated by applying our diffusion rendering method using several paper models. Note the remarkable differences solely due to varying the paper type. The body of the dragon fly was drawn with ink containing only a limited amount of water, whereas the wings were drawn using ink containing abundant amount of water such that they more visibly show diffusion effects. In addition, at the end of the wings where two strokes overlap, the image shows a realistic combination of diffusion intensity, being obtained by calculating the background and foreground image intensity. The right wings drawn with an abundant amount of water are controlled to lighten the color intensity of the body; thereby showing the correct perspective relationship between the objects.

The proposed ink diffusion rendering method possesses several computational merits, i.e.,

- The user can select from a wide variety of paper textures consisting of randomly distributed fiber mesh and uniform fabric mesh.
- The computational complexity of wave schema is linear: \( O(n) \).
- Realistic diffusion images are generated by considering both the global change and local variation of image intensity peculiar to the selected paper type.
- When coupled with the soft brush [17], all operations such as inputting of strokes, selecting paper type, and specifying parameter values can be carried out interactively and naturally, without editing 2D control points.

Drawing the diffusion image of the dragon fly painting (Fig. 17) took an average of just 5 s using a SGI O2 system; a speed sufficient for users to believe that diffusion occurs in real time. Such reasonable performance is due to the employed flow schema that works in linear time by preventing ink from flowing backward.

One general rule in actual oriental black ink painting is that strokes are performed only once, i.e., there can be no later altering, touching up, or adding to them; unlike oil painting where mistakes may be scraped off or painted over. With computer-based painting, however, painters can try the same brush stroke as many times as needed to obtain the proper effect, and doing so without wasting paper. When considering that a single improper stroke can ruin an entire painting, this advantage is especially beneficial.

There exist other interesting rendering effects in black ink painting that have not been discussed here. One notable feature is scratchiness (kasure in Japanese), being the white scratchy texture displayed when the brush moves fast or the ink runs out in the middle of a stroke. This feature is affected by many physical factors such as pressure applied on the brush, speed of moving a brush, and the roughness of the paper surface. Development of required physical models and algorithms is left for future research.

References


